AP® Computer Science A Elevens Lab Student Guide

The AP Program wishes to acknowledge and thank the following individuals for their contributions in developing this lab and the accompanying documentation.
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Activity 3: Shuffling the Cards in a Deck

Introduction:

Think about how you shuffle a deck of cards by hand. How well do you think it randomizes the cards in the deck?

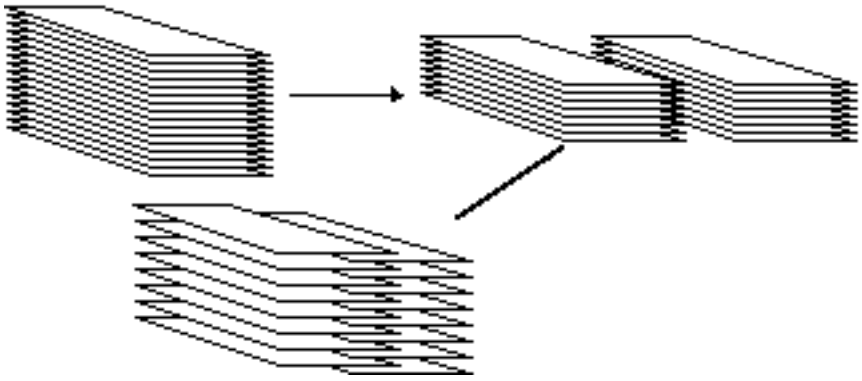
Exploration:

We now consider the *shuffling* of a deck, that is, the *permutation* of its cards into a random-looking sequence. A requirement of the shuffling procedure is that any particular permutation has just as much chance of occurring as any other. We will be using the `Math.random` method to generate random numbers to produce these permutations.

Several ideas for designing a shuffling method come to mind. We will consider two:

Perfect Shuffle

Card players often shuffle by splitting the deck in half and then interleaving the two half-decks, as shown below.



This procedure is called a *perfect shuffle* if the interleaving alternates between the two half-decks. Unfortunately, the perfect shuffle comes nowhere near generating all possible deck permutations. In fact, eight shuffles of a 52-card deck return the deck to its original state!

Consider the following "perfect shuffle" algorithm that starts with an array named `cards` that contains 52 cards and creates an array named `shuffled`.

> Initialize `shuffled` to contain 52 "empty" elements.
> Set `k` to 0.
> For `j` = 0 to 25,
> - Copy `cards[j]` to `shuffled[k]`;
> - Set `k` to `k+2`.
>
> Set `k` to 1.
> For `j` = 26 to 51,
> - Copy `cards[j]` to `shuffled[k]`;
> - Set `k` to `k+2`.

This approach moves the first half of `cards` to the even index positions of `shuffled`, and it moves the second half of `cards` to the odd index positions of `shuffled`.

The above algorithm shuffles 52 cards. If an odd number of cards is shuffled, the array `shuffled` has one more even-indexed position than odd-indexed positions. Therefore, the first loop must copy one more card than the second loop does. This requires rounding up when calculating the index of the middle of the deck. In other words, in the first loop `j` must go up to `(cards.length + 1) / 2`, exclusive, and in the second loop `j` most begin at `(cards.length + 1) / 2`.

Selection Shuffle

Consider the following algorithm that starts with an array named `cards` that contains 52 cards and creates an array named `shuffled`. We will call this algorithm the "selection shuffle."

> Initialize `shuffled` to contain 52 "empty" elements.
> Then for `k` = 0 to 51,
> - Repeatedly generate a random integer `j` between 0 and 51, inclusive until `cards[j]` contains a card (not marked as empty);
> - Copy `cards[j]` to `shuffled[k]`;
> - Set `cards[j]` to empty.

This approach finds a suitable card for the k^{th} position of the deck. Unsuitable candidates are any cards that have already been placed in the deck.

While this is a more promising approach than the perfect shuffle, its big defect is that it runs too slowly. Every time an empty element is selected, it has to loop again. To determine the last element of `shuffled` requires an average of 52 calls to the random number generator.

A better version, the "efficient selection shuffle," works as follows:

> For `k` = 51 down to 1,
> - Generate a random integer `r` between 0 and `k`, inclusive;
> - Exchange `cards[k]` and `cards[r]`.

This has the same structure as selection sort:

> For `k` = 51 down to 1,
> - Find `r`, the position of the largest value among `cards[0]` through `cards[k]`;
> - Exchange `cards[k]` and `cards[r]`.

The selection shuffle algorithm does not require to a loop to find the largest (or smallest) value to swap, so it works quickly.

Exercises:

1. Use the file `Shuffler.java`, found in the **Activity3 Starter Code**, to implement the perfect shuffle and the efficient selection shuffle methods as described in the **Exploration** section of this activity. You will be shuffling arrays of integers.

2. `Shuffler.java` also provides a `main` method that calls the shuffling methods. Execute the `main` method and inspect the output to see how well each shuffle method actually randomizes the array elements. You should execute `main` with different values of `SHUFFLE_COUNT` and `VALUE_COUNT`.

Questions:

1. Write a static method named `flip` that simulates a flip of a weighted coin by returning either `"heads"` or `"tails"` each time it is called. The coin is twice as likely to turn up heads as tails. Thus, `flip` should return `"heads"` about twice as often as it returns `"tails."`

2. Write a static method named `arePermutations` that, given two `int` arrays of the same length but with no duplicate elements, returns `true` if one array is a permutation of the other (i.e., the arrays differ only in how their contents are arranged). Otherwise, it should return `false`.

3. Suppose that the initial contents of the `values` array in `Shuffler.java` are `{1, 2, 3, 4}`. For what sequence of random integers would the efficient selection shuffle change `values` to contain `{4, 3, 2, 1}`?

Activity 4: Adding a `Shuffle` Method to the `Deck` Class

Introduction:

You implemented a `Deck` class in Activity 2. This class should be complete except for the `shuffle` method. You also implemented a `DeckTester` class that you used to test your incomplete `Deck` class.

In Activity 3, you implemented methods in the `Shuffler` class, which shuffled integers.

Now you will use what you learned about shuffling in Activity 3 to implement the `Deck shuffle` method.

Exercises:

1. The file `Deck.java`, found in the **Activity4 Starter Code** folder, is a correct solution from Activity 2. Complete the `Deck` class by implementing the `shuffle` method. Use the efficient selection shuffle algorithm from Activity 3.

   Note that the `Deck` constructor creates the deck and then calls the `shuffle` method. The `shuffle` method also needs to reset the value of `size` to indicate that all of the cards can be dealt again.

2. The `DeckTester.java` file, found in the **Activity4 Starter Code** folder, provides a basic set of `Deck` tests. It is similar to the `DeckTester` class you might have written in Activity 2. Add additional code at the bottom of the `main` method to create a standard deck of 52 cards and test the `shuffle` method. You can use the `Deck toString` method to "see" the cards after every shuffle.

Glossary

assertion: Boolean expressions that should be true if the program is running correctly. The Java `assert` statement can be used to check assertions in a program.

class invariant: A logical statement relating to the values of the instance variables of a class that is always true between calls to the class's methods (also referred to as a "data invariant"). ("Invariant" means "not varying" or "not changing.")

client class: A class that uses another class (e.g., The `Deck` class is a client of the `Card` class.).

helper method: A method, usually `private`, that is called by another method. Helper methods are used to simplify the calling method. They also facilitate code reuse when they provide a function that can be used by more than one calling method.

loop invariant: A logical statement that is always true when execution reaches a loop's termination test.

model: A class with behaviors and state that represent key features of some "real-world" object or process. We say that a class models the "real-world" object. For example, the `Deck` class models a real deck of cards.

perfect shuffle: A card-shuffling method that starts with dividing the deck into two stacks, then interleaving the cards, first a card from stack 1, then a card from stack 2, then another card from stack 1, another from stack 2, and so on.

permutation: A rearrangement of a given sequence of values. There are six permutations of the sequence [1,2,3], namely [1,2,3] (the "identity" permutation), [1,3,2], [2,1,3], [2,3,1], [3,1,2], and [3,2,1]. If the given sequence contains duplicate values, so will its permutations. For example, the permutations of [1,1,2] are [1,1,2], [1,2,1], and [2,1,1].

polymorphism: A process that Java uses where the method to execute is based on the object executing the method. For example, if `board.anotherPlayIsPossible()` is executed, and `board` references an `ElevensBoard` object, then the `ElevensBoard` `anotherPlayIsPossible` method will be called.

probabilistic: Based on chance or involving the use of randomness.

pseudo-random number generator: A procedure that produces a sequence of values that passes various statistical tests for randomness (e.g., any value is just as likely to occur in a given position in the sequence as any other).

random number generator: See **pseudo-random number generator**.

refactor: Reorganizing code. One example of refactoring is creating helper methods to simplify code or eliminate duplicate code. Another is splitting a class into a superclass and a subclass, putting the code that would be common to other subclasses into the new superclass.

selection shuffle: A card-shuffling method that works similarly to the selection sort. It randomly selects a card for each position in the deck from the remaining unselected cards.

shuffle: A method of permuting (mixing up) the cards in a deck. See **perfect shuffle** and **selection shuffle**.

simulation: Imitation, using a computer program, of some real-world process. The "actors" in the process correspond to objects and variables in the simulation, while the interactions between the actors correspond to program methods.

systematic: Performed using a logical step-by-step process.

truncation: Removal of the fractional part of a real or double value, producing an integer.

References

The Complete Book of Solitaire and Patience Games, by Albert H. Morehead and Geoffrey Mott-Smith, Bantam Books (1977).